Section 13

Work

Work problems pretty much live up to their title. They involve two people, two machines, or two objects working together to complete a job.

There are two basic types of work problems. One type involves two people (or any two objects) working together to complete a job. The other type deals with a container that is being both filled and emptied at the same time.

Step 1
Read Through The Entire Problem

First you need to determine which type of work problem it is in order to know which equation setup to use.

- If the problem involves two people (or any two objects) working together to complete a job, that information will be clearly stated in the problem. You need to look for and take note of the time it takes each person working alone to complete the job, and how long it takes them if they work together.

- If the problem is the type that deals with a container that is being filled and emptied, that will be also be clearly stated in the problem. With this second type of work problem, you need to look for and take note of the rate of time it takes to fill the container and the rate of time it takes to empty the container.

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Step 2
Naming The Expressions

When the Work Problem involves two people (or any two objects) working together to complete a job, your equation is set up using two rational expressions (fractions). Each fraction will be "time together" over "time alone".

- The "time together" is the time it takes the two people working together to complete the job. It may be given as an amount or it may be an unknown. Either way, it will always be the same in both fractions.

- The "time alone" is the amount of time it takes one person working alone to finish the job. The "time alone" will be different for each fraction, and will be either two given amounts, one given amount and one unknown, or two unknowns that you will name expressions for using Direct Translation.

When the work problem is the type that deals with a container that is being filled and emptied, you also set up two fractions.

- The first fraction will be the "time together" over the "filling time". The second fraction will be the "time together" over the "emptying time".
- In this type of problem, the "time together" represents the total amount of time it takes to fill a container when it is being filled and emptied *at the same time*.
- The "time together" will always be the same for both fractions, and will always be an unknown.
- The "filling time" and the "emptying time" will be given in the problem.

Two People Working Together	"*Time Together*" is the amount of time it takes both people working together to complete the job.
Filling & Emptying A Container	"*Time Together*" is the amount of time it takes to fill the container when the container is being filled and emptied at the same time.

Step 3

Set Up An Equation

When the work problem is the type that has two people (or any two objects) working together to complete a job, each fraction represents the portion of the job that one person has completed. The equation you use is the two fractions added together and set equal to "1". The number "1" is used because the fractions represent each person's part of the job and the number "1" represents one whole job completed.

When the work problem is the type that involves a container that is being filled and emptied, the first fraction represents the part when the container is being filled and the second fraction represents the part when the container is being emptied. To set up the equation, the 2nd fraction (the "emptying time") is *subtracted* from the 1st fraction (the "filling time") and set equal to "1". The number "1" is used because the "filling time" fraction less the "emptying time" fraction represents one whole task completed.

Type Of Work Problem	**Equation To Use**
Two People (Objects) Working Together	$\frac{\text{time together}}{\text{time alone}} + \frac{\text{time together}}{\text{time alone}} = 1$
Filling And Emptying A Container	$\frac{\text{time together}}{\text{filling time}} - \frac{\text{time together}}{\text{emptying time}} = 1$

Step 4
Solve the Equation

Using the method taught by your instructor, solve the equation for the variable. Keep in mind when solving a Work Problem, you may get two solutions to the equation. If one of these solutions is negative, eliminate it because an amount of time **cannot be negative**.

Step 5
Make Sure to Answer the Question Being Asked

You need to make sure what question is being asked in the problem. It is possible that the value for the variable x may be your answer. But it may *not* be.

The solution to the equation will be the answer to the question if there is no Direct Translation used to name expressions. If Direct Translation is used, you need to make sure of what the question is asking. To get the answer, you may need to substitute the value of x into the expression you named in Step 2.

EXAMPLES

EXAMPLE 1 Steve can decorate a classroom in 3 hours. It takes Joel 4 hours to decorate the same classroom. How long will it take them to decorate the classroom working together?

SOLUTION

Step 1 *Read Through The Entire Problem*

- The problem involves two people working together.
- Steve can complete the job in 3 hours, working alone.
- Joel can complete the job in 4 hours, working alone.
- The time it takes them working together to complete the job is unknown.

Step 2 *Name The Expressions*

- The fractions will be "time together" over "time alone".
- The "time together" is unknown and will be represented by x in both fractions.
- The "time alone" for the 1st fraction representing Steve's time is given. It is 3.
- The "time alone" for the 2nd fraction representing Joel's time is given. It is 4.

Step 3 *Set Up The Equation*

- The equation used is 1st fraction plus 2nd fraction equals 1.
- The 1st fraction representing Steve's time is x over 3.
- The 2nd fraction representing Joel's time is x over 4.

$$\frac{x}{3} + \frac{x}{4} = 1$$

Step 4 *Solve The Equation*

- The solution to the equation is $x = 1\frac{5}{7}$

Step 5 *Answer The Question Asked*

- You have the solution to the equation.
- The question asks the time it takes both of them to complete the job working together.
- The value of x is the time it takes them to complete the job working together.
- You are done. You have the correct answer.

Answer: It takes them $1\frac{5}{7}$ hours to decorate the classroom working together.

EXAMPLE 2 The time it takes Elena to clean the kitchen is twice as long as the time it takes Camelia. Working together, they can clean the kitchen in 2 hours. How long would it take Elena to clean the kitchen working by herself?

SOLUTION

Step 1 *Read Through The Entire Problem*

- The problem involves two people working together.
- Elena takes twice the amount of time to complete the job as Camelia.
- The time it takes them working together to complete the job is 2 hours.

Step 2 *Name The Expressions*

- The fractions will be "time together" over "time alone".
- The "time together" is given. It is 2, and will be used in the numerator for both fractions.
- Use Direct Translation to determine the expressions for "time alone" for each fraction.
- The "time alone" it takes Camelia to do the work is unknown. Use the variable x.
- The "time alone" it takes Elena to do the work is twice as long as Camelia. Use $2x$.

Step 3 *Set Up The Equation*

- The equation used is 1st fraction plus 2nd fraction equals 1.
- The 1st fraction representing Camelia's time is 2 over x.
- The 2nd fraction representing Elena's time is 2 over $2x$.

$$\frac{2}{x} + \frac{2}{2x} = 1$$

Step 4 *Solve The Equation*

- The solution to the equation is $x = 3$

Step 5 *Answer The Question Asked*

- You have the solution to the equation, but it is not the answer.
- The question asks the time it takes Elena to complete the job working alone.
- The value of x is the time it takes Camelia to complete the job working alone.
- Substitute the solution for x (3) into the expression you named for Elena's time.

Elena's Time = $2x$
Elena's Time = $2(3)$
Elena's Time = 6

Answer: It takes Elena 6 hours by herself.

EXAMPLE 3 Using a garden hose, it takes 15 minutes to fill up a child's wading pool. It takes 45 minutes to empty out the pool through a small drainage hole. If the drainage hole is left open by mistake, and the water is draining out at the same time the pool is being filled with the garden hose, how long will it take before the pool is filled up?

SOLUTION

Step 1 *Read Through The Entire Problem*

- The problem involves a container being filled and emptied.
- It takes 15 minutes to fill up the pool.
- It takes 45 minutes to empty out the pool.

Step 2 *Name The Expressions*

- The 1st fraction will be "time together" over "filling time".
- The 2nd fraction will be "time together" over "emptying time".
- The "time together" is always unknown. Use x as the numerator for both fractions.
- The "filling time" of the 1st fraction is given. It is 15 minutes.
- The "emptying time" of the 2nd fraction is given. It is 45 minutes.

Step 3 *Set Up The Equation*

- The equation used is 1st fraction (filling) minus 2nd fraction (emptying) equals 1.
- The 1st fraction representing the filling time is x over 15.
- The 2nd fraction representing emptying time is x over 45.

$$\frac{x}{15} - \frac{x}{45} = 1$$

Step 4 *Solve The Equation*

- The solution to the equation is $x = 22\frac{1}{2}$

Step 5 *Answer The Question Asked*

- You have the solution to the equation.
- There is no Direct Translation used to name the expressions.

- The solution to the equation is the answer to the question that is asked.
- You are done. You have the correct answer.

Answer: It will take $22\frac{1}{2}$ minutes to fill up the pool.

Work Problems: Exercise Set

1. Beavis can wash the walls in 3 hours working alone, and Bart can wash the walls in 2 hours. How long would it take them to wash the walls if they worked together?

2. Eric can install the carpet in a room in 3 hours, but Mark needs 5 hours. How long will it take them to complete the job if they work together?

3. Working alone, it takes Jacob 5 minutes longer to wash the dishes than it takes Sarah when she does it alone. Washing the dishes together, Jacob and Sarah can finish the dishes in 6 minutes. How long does it take Jacob to wash the dishes by himself?

4. It takes Vinnie 2 hours to groom a poodle by himself. Marie can groom the same poodle in 4 hours. How long would it take them if they worked together to groom the poodle?

5. Working together, Benjamin and Alex can paint a room in 4 hours. Working alone, it would take Benjamin 7 hours to paint the room. How long would it take Alex to paint the room working alone?

6. Working together, Michelle and Lauren can make a quilt in 5 days. If it takes Michelle 8 days to make a quilt by herself, how long would it take Lauren to make a quilt by herself?

7. It takes an older inkjet printer three times as long as it takes a new laser printer to print out a math workbook. With both printers running, the workbook can be printed out in 9 hours. How long would it take the old printer to print out the whole workbook by itself?

8. It takes Jarred 3 minutes longer than Sierra to eat a chocolate bar. If it takes them 2 minutes to eat the same chocolate bar together, how long does it take Jarred to eat the chocolate bar by himself?

9. It takes Luke 9 hours longer than Laura to do the store's inventory. If they can finish the inventory in 6 hours working together, how long does it take Laura to do the inventory alone?

10. It takes Ashley 3 times longer than Taylor to landscape their backyard. Working together, they can they can finish the job in 21 days. How long does it take Ashley to landscape the backyard if she works alone?

11. It takes Mr. Spock 30 minutes to fight off a Klingon warship by himself. Captain Kirk can fight them off in 45 minutes. How long will it take them to fight off the Klingons if they fight together?

12. Don can write a research paper in 7 days. His girlfriend, Adrienne, can write a research paper in 4 days. If the instructor assigned this as a team project, how long would it take Don and Adrienne if they write the research paper together?

13. Ramses and Moses could build a pyramid in 3 months. If it takes Ramses 5 months to build a pyramid by himself, how long would it take Moses to build a pyramid by himself?

14. It takes Jerry 18 more minutes than Dean to wash a car. If they can wash the car together in 12 minutes, how long does it take Jerry to wash the car by himself?

15. On Nickelodeon's Double Dare Show, a large vat of green goop can be filled by a hose in 12 hours. The vat can be emptied by a different hose in 15 hours. How long will it take to fill the vat with goop if both the filling hose and the emptying hose are both being used?

16. It takes 9 minutes to fill up the kitchen sink with water from the faucet, and it takes 12 minutes to drain the water out. How long will it take to fill up the sink if the drain is left open.

17. It takes 10 minutes to fill a water cooler with filtered water and 12 minutes to empty the water cooler with the drain plug open. Donovan tried to fill up the water cooler without realizing that the drain plug had been left open. How long did it take Donovan to fill up the water cooler?

18. It takes 20 hours to fill up a cement-mixing truck with cement and 25 hours to empty out the truck. If the cement is pouring out of the truck at the same time it is being poured into the truck, how long will it take to completely fill the truck with cement?

19. One pipe can fill an oil tanker in 5 hours and another pipe can empty the oil tanker in 10 hours. If both pipes are in use at the same time, how long will it take to fill up the oil tanker?

20. It takes 4 minutes to fill up a bucket of paint. If someone pokes a large hole in the bottom of the bucket, the paint will empty out in 6 minutes. If a painter tries to fill up this bucket with the hole in the bottom, how long will it take him?